Vacuum Saturation Hypothesis and QCD Sum Rules[1]

K.G. Chetyrkin and A.A. Pivovarov

Institute for Nuclear Research of the Academy of Sciences of Russian Federation,
60th October Anniversary prospect, 7a, 117312 Moscow, Russian Federation

Abstract

The accuracy of the vacuum saturation hypothesis is discussed using the examples of vacuum expectation values of four-quark operators and the parameter B, which determines the short-distance contribution to the $K^0 - \bar{K}^0$ mixing.

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[1]Copy of the paper published in *IL NUOVO CIMENTO*, **100 A** (1988) 899. Received 28 March 1988. Some comments, related to a (relatively small) change of the updated result due to changes in the values of incorporated phenomenological parameters, are added.

1. - The standard model of strong and electroweak interactions predicts the existence of transitions changing the strangeness by two units. These processes are due to exchange of intermediate vector bosons and lead to a possibility of $K^0 - \bar{K}^0$ mixing. A calculation of the corresponding effective Hamiltonian on the basis of the standard model was first performed by Gaillard and Lee [1] by expanding the interaction in the inverse mass of the intermediate boson. The comparison of the result of this calculation with the experimentally observed mass difference of K_L and K_S mesons led to a prediction for the c-quark mass before experimental detection of J/ψ particle. In spite of the considerable theoretical uncertainty, involved in evaluation of the matrix element $\langle K^0|H_{eff}|\bar{K}^0\rangle$ the predicted quark mass happened to be in rather good agreement with the measured one. However, the value of the $K_L - K_S$ mass difference is quite sensitive to the numerical values of such parameters of the standard model as the masses of the t- and c-quarks, the mixing angles between various quark generations and the general structure of the model. Since the pioneer calculation by Gaillard and Lee, a good deal of work has been carried out to refine its results.

Account was taken of the strong-interaction corrections to H_{eff} in the leading logs approximation and for the three-quark generations by Gillman and Wise [2]. The obtained expression for the $K_L - K_S$ mass difference is of the form

$$\Delta m = m_L - m_S = 2\mathrm{Re}\, M_{12}$$

where

$$M_{12} = \frac{G_F^2 M_W^2}{4\pi^2}\left\{\lambda_c^2\eta_1\frac{m_c^2}{M_W^2} + \lambda_t^2\eta_2\frac{m_t^2}{M_W^2} + 2\lambda_c\lambda_t\frac{m_c^2}{M_W^2}\ln\frac{m_t^2}{M_W^2}\right\}\left(\frac{\alpha_s^{''}(m_c^2)}{\alpha_s^{'''}(\mu^2)}\right)^{2/9}\frac{\langle\bar{K}^0|\hat{O}|K^0\rangle}{2m_K}. \quad (1)$$

Here the standard notations were used: $\lambda_i = V_{id}V_{is}^*$, $i = u, c, t$ being the elements of the Kobayashi-Maskawa matrix; η_i's are some coefficients due to strong-interaction contributions, $\hat{O}$ is the four-quark operator $\hat{O} = (\bar{s}_L\gamma_\mu d_L)^2$ with $\Delta S = 2$.

Equation (1) was obtained with the help of the Wilson expansion at short distances, so this contribution to the mass difference Δm should be more correctly called the contribution of short distances. Actually, the existence of the other contributions to the mass difference was demonstrated by Wolfenstein [3]. These new contributions are essentially different from those under discussion and come from long distances. We shall not dwell upon them and reserve the term "mass difference" and notation Δm for the short-distance contribution only.

The strong interactions enter eq. (1) through Wilson's coefficient η_i's, which can be reliably calculated because of the property of the asymptotic freedom of QCD, and through the matrix element of the four-quark operator $\hat{O}$, being responsible for the $\Delta S = 2$ transitions. It is this matrix element we are going to discuss. It should be stressed that this problem is of gravely nonperturbative nature and, thus, it is a hard nut to crack by any of the existing techniques. However, because of the great importance of the numerical value of the matrix element several attempts have been undertaken to evaluate it.

Gaillard and Lee have estimated this matrix element by means of the vacuum saturation hypothesis with the result

$$\langle\bar{K}^0|\hat{O}|K^0\rangle^{VS} = \frac{2}{3}f_K^2 m_K^2,$$

f_K being the K-meson decay constant. It is convenient to parameterized the exact matrix element with the help of dimensionless parameter B as follows:

$$\langle \bar{K}^0|\hat{O}|K^0\rangle = B\langle \bar{K}^0|\hat{O}|K^0\rangle^{VS}. \tag{2}$$

A lot of efforts has been made to bypass the vacuum saturation hypothesis and to calculate the parameter B independently, the obtained results diverge from each other [4-12].

This work aims at giving an analysis of attempts to compute B within the QCD sum rules approach.

First, Chetyrkin et al. [12] applied the technique of the QCD sum rules to the three-point correlator comprising the operator $\hat{O}$ and the interpolating currents of the K-mesons with the result $\hat{B} = 1.2 \pm 0.1$. Second, after completion of that work there appeared similar in its spirit the calculation by Pich and Rafael [10]. They used the two-point correlator of the four-quark operators and obtained $B = 0.38 \pm 0.09$.

It is of interest to trace the cause of this marked difference since the sum rules method, as a rule, works with an accuracy of about $(20 \div 30)\%$.

The work is organized as follows. In sect. 2 we briefly describe the calculation of B by means of a three-point correlator and discuss the accuracy of the vacuum saturation hypothesis employing there to estimate the vacuum expectation values (v.e.v.) of four-quark operators. Section 3 is devoted to a comparison of this calculation with the one performed by Pich and Rafael. Section 4 contains our conclusions.

2. - The starting point of the work [12] is the following representation:

$$\langle \bar{K}^0|\hat{O}|K^0\rangle = \lim_{p^2\to m_K^2}\lim_{K^2\to m_K^2}(p^2-m_K^2)(K^2-m_K^2)p^\mu K^\nu \tag{3}$$

$$m_K^{-4}f_K^{-2}i^2\int \mathrm{d}x\,\mathrm{d}y\,\langle 0|Tj_\mu^5(x)j_\nu^5(y)\hat{O}(0)|0\rangle \exp[ipx-iKy],$$

which can be obtained with the reduction formulae. Thus, to find the matrix element (3) one needs to compute the function

$$T_{\mu\nu}(p,q) = i^2\int \mathrm{d}x\,\mathrm{d}y\,\exp[ipx-iqy]\langle Tj_\mu^5(x)\hat{O}(y)j_\nu^5(0)\rangle = \tag{4}$$

$$= p_\mu q_\nu T(p^2,(p-q)^2,q^2) + \text{other structures},$$

where $j_\mu^5 = \bar{d}\gamma_\mu\gamma_5 s$ is the interpolating field of the K^0-meson:

$$\langle 0|j_\mu^5(0)|K^0(p)\rangle = ip_\mu f_K, \quad f_K = 1.17 f_\pi.$$

The function (4) can be reliably calculated at small q and large Euclidean $p^2 > 1$ GeV^2 by means of the (somewhat modified) operator product expansion technique [13]. On the other hand, the matrix element (2) is connected with the amplitude $T(-t,-t,0) \equiv T(t)$ by the dispersion relation

$$T(t) = \int ds\frac{\rho(s)}{s+t} - \text{subtractions} = f_K^2\frac{\langle \bar{K}^0|\hat{O}|K^0\rangle}{(t+m_K^2)^2} + \frac{A}{t+m_K^2} + \ldots, \tag{5}$$

where the one-pole contribution corresponds to transitions of the K^0-meson to other (different from K^0) states and dots stand for the higher-state contributions.

Within the vacuum saturation approximation the function $T_{\mu\nu}(p,q)$ assumes the form

$$T^{VS}_{\mu\nu} = \frac{8}{3}\Pi_{\mu\alpha}(p)\Pi_{\nu\alpha}(p-q), \quad \Pi_{\mu\alpha}(p) = i\int \mathrm{dx}\ \exp[ipx]\langle Tj^5_\mu(x)\bar{s}_L(0)\gamma_\alpha d_L(0)\rangle \tag{6}$$

and the resulting value of B proves to be $B^{VS} = 1$. Thus, there remains to compute only the function $\Delta_{\mu\nu} = T_{\mu\nu} - T^{VS}_{\mu\nu}$, which is responsible for all the departures from the vacuum saturation prediction for B.

The computed result for $\Delta_{\mu\nu}$ is (only local operators with dimension ≤ 6 were taken into account) [12]

$$\Delta_{\mu\nu}(p,q) = p_\mu q_\nu\left\{-5(pq)\frac{\langle\alpha_s G^2\rangle}{192\pi^3} - 4\langle\bar{d}s\bar{s}d\rangle - 4\langle\bar{d}d\bar{s}s\rangle + \right. \tag{7}$$

$$\left. +2\langle\bar{s}s\bar{s}s\rangle + 2\langle\bar{d}d\bar{d}d\rangle + \frac{m_s\langle g\bar{d}G_{\mu\nu}\sigma_{\mu\nu}d\rangle}{24\pi^2}\right\}\frac{1}{p^2(p-q)^2} + \text{other structures,}$$

where the designation $\langle\bar{d}s\bar{s}d\rangle$ stands for $\langle\bar{d}_L\gamma^\alpha s_L\bar{s}_L\gamma^\alpha d_L\rangle$ and so on; $G^2 = G^a_{\mu\nu}G^a_{\mu\nu}$; $G_{\mu\nu} = G^a_{\mu\nu}t^a$; $tr(t^a t^b) = (1/2)\delta^{ab}$; $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$.

On putting $q = 0$ and using the method of finite energy sum rules [14] to connect the phenomenological (eq. (5)) and the theoretical (eq. (7)) representations of the function $T(t)$, one gets

$$\frac{2}{3}f^4_K m^2_K(B-1) = \int_0^{s_0}\rho^{th}(s)(s+m^2_K)ds = (24\pi^2)^{-1}m_s\langle g\bar{d}\sigma_{\mu\nu}G_{\mu\nu}d\rangle$$

or

$$B - 1 = \frac{m_s\langle g\bar{d}\sigma_{\mu\nu}G_{\mu\nu}d\rangle}{16\pi^2 f^4_K m^2_K} \tag{8}$$

To estimate the v.e.v.'s of four-quark operators appearing in eq. (7) the hypothesis of vacuum saturation has been used, which leads to the vanishing of each of these v.e.v.'s. Assuming the relation [15]

$$\langle g\bar{d}\sigma_{\mu\nu}G_{\mu\nu}d\rangle = m^2_0\langle\bar{d}d\rangle\ , \quad m^2_0 = (0.8 \pm 0.4)\ \text{GeV}^2,$$

one finds for the renormalization group invariant quantity [2]

$$\hat{B} = B(\mu)(\alpha_s(\mu))^{-2/9} = 1.2 \pm 0.1. \tag{9}$$

This estimate is in good agreement with the hypothesis of vacuum saturation; within this approach this means the absence of operators giving appreciable contributions to $\Delta_{\mu\nu}$.

[2]Having in mind a solid theoretical basis of our calculation we have just updated the prediction for $\hat{B}_K$ by using present values of relevant parameters. The quantity $B-1$ in eq. (8) is expressed through well known parameters that did not change much during last years. The bulk of the change of the parameter $\hat{B}$ is due to the normalization (the change in the factor $\alpha_s(1.2\ \text{GeV}^2)^{-2/9}$ in eq. (9)). The updated version of eq. (9) with $\alpha_s(1.2\ \text{GeV}^2) = 0.69$ reads now:

$$\hat{B} = 1.0 \pm 0.1$$

In principle there may exist three kinds of extra corrections to $\Delta_{\mu\nu}$:

i) corrections of higher order in α_s;

ii) corrections due to bilocal operator;

iii) corrections due to local operators.

Let us consider them in turn. The first contribution is suppressed by the factor $\alpha_s/\pi \sim 0.1$ and can hardly change the result (9) to a large extent. The second one is suppressed by an extra power of p^{-2} and does not contribute at all within the finite energy sum rules approach. (This statement is valid as long as one neglects, as we always do, a weak logarithmic dependence on p^2 induced by anomalous dimensions of the operators involved.) The third type of terms have been taken into account in eq. (7) in the leading order with a small total effect on $(B-1)$. Thus, all the contributions are under control within the approach and prove to be small.

However, there exists another source of uncertainty – the use of the procedure of vacuum saturation to evaluate the v.e.v.'s of four-quark operators, which might (as simple estimates show) violate drastically eq. (9). Let us discuss the issue in some detail.

To begin with, it is easy to show that the peculiar combination of four-quark operators entering eq. (7) transforms as a member of a 27-plet with respect to the (flavour) $SU^f(3)$ group. This means that the corresponding v.e.v's might only appear in the second order of the expansion in the strange-quark mass. It can be checked that this suppression is still operative if one takes into account next-to-leading corrections to the coefficient functions of the operators under consideration. However, it is interesting to get a direct estimate of the accuracy of the vacuum saturation hypothesis. To this end it is convenient to employ the QCD sum rules method in configuration space (x-space). It will be seen below that such x-space sum rules provide the unique possibility of direct estimation of the v.e.v.'s of four-quark operators.

Indeed, let us consider the correlator

$$\langle T j^\mu(x) j_\mu(0)\rangle = \Pi(-x^2),$$

where $j^\mu = \bar{u}\gamma^\mu d$ is the interpolating current of the ρ-meson. At $x^2 \to 0$ the following x-space operator expansion holds:

$$\Pi(-x^2) = \frac{6}{\pi^4 x^6} + \frac{\langle \alpha_s G^2\rangle}{16\pi^3 x^2} + \langle j^\mu j_\mu\rangle + \frac{\alpha_s}{4\pi}\left[6\left(L + 2\gamma_E + \frac{1}{2}\right)\langle \bar{u}\gamma_5\gamma^\lambda t^a d\bar{d}\gamma_5\gamma^\lambda t^a u\rangle + \right. \tag{10}$$

$$\left. + \left(\frac{2}{3}L + \frac{4}{3}\gamma_E - \frac{13}{9}\right)\langle \bar{\psi}\gamma^\lambda t^a \psi(\bar{u}\gamma^\lambda t^a u + \bar{d}\gamma^\lambda t^a d)\rangle\right] + o(1)$$

where $L = \ln(-\mu^2 x^2/4)$, $\gamma_E = 0.577\ldots$, and μ is the normalization point of the $\overline{\mathrm{MS}}$-scheme. Expansion (10) can be obtained from the corresponding one in p-space. Note that the values of the constants added to L in the right-hand side of eq. (10) are fixed by the recipe for renormalization of the operator $j^\mu j_\mu$. Note also that expansion (10) remains valid after substitutions $j^\mu \to j_5^\mu$, $\gamma_5\gamma_\lambda \to \gamma_\lambda$, which correspond to considering the axial current interpolating the A_1-meson.

On the other hand the following Kallen-Lehmann representation takes place:

$$\Pi(-x^2) = \int_{4m_\pi^2}^{\infty} \rho(s)\Delta^c(-x^2,s)ds, \quad \Delta^c(x^2,s) = \frac{1}{4\pi^2 x^2}\sqrt{x^2 s}\,K_1(\sqrt{x^2 s}), \tag{11}$$

where the spectral density can be roughly approximated as follows:

$$\rho(s) = F\delta(s-m^2) + a\theta(s-s_0),$$

the parameters F, m, s_0, a being known from the analysis of the same correlator in p-space [16].

The main advantage of relations (10) and (11) over similar sum rules in p-space comes from the fact that the contribution of the operator $j^\mu j_\mu$ on the right-hand side of eq. (10) is parametrically increased due to the contact term pictured in fig. 1 and corresponding to a disconnected diagram. This allows one to use eqs. (10) and (11) for determining the v.e.v. $\langle j^\mu j_\mu\rangle$ directly.

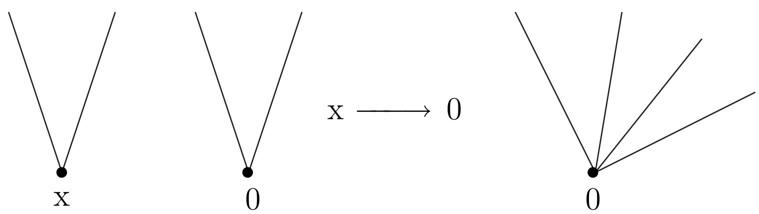

Fig. 1 - The origin of the contact terms in the x-space operator product expansion.

An analysis of sum rules in x-space leads to the relations (see fig. 2 and 3; we took $\mu = 1$ GeV, $\alpha_s(1\,\mathrm{GeV}) = 0.3$, $\langle\bar{q}q\rangle = (-0.23\ \mathrm{GeV})^3$)

$$\langle\bar{u}\gamma^\mu d\bar{d}\gamma^\mu u\rangle/\langle\bar{q}q\rangle^2 = -\frac{1}{3}(0.90 \pm 0.15)\,,$$

$$\langle\bar{u}\gamma^\mu\gamma_5 d\bar{d}\gamma^\mu\gamma_5 u\rangle/\langle\bar{q}q\rangle^2 = \frac{1}{3}(0.84 \pm 0.20)\,,$$

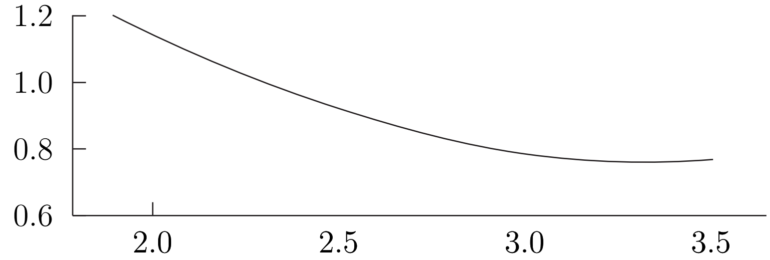

Fig. 2 - The curve showing the dependence of the numerical value of the v.e.v. $\langle \bar{u}\gamma^\mu \bar{d} d\gamma^\mu u\rangle$ on the variable $|x| = \sqrt{-x^2}$.

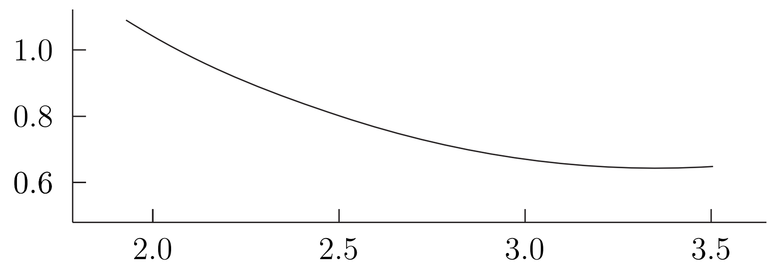

Fig. 3 - The curve showing the dependence of the numerical value of the v.e.v. $\langle \bar{u}\gamma^\mu\gamma_5 d\bar{d}\gamma^\mu\gamma_5 u\rangle$ on the variable $|x| = \sqrt{-x^2}$.

which are in agreement with the result of the vacuum saturation within an accuracy 30%. The relative discrepancy between v.e.v.'s of four-quark operators of VV and AA types is about 10%; this fact indicates in favor of a small contribution of four-quark operators entering the right-hand side of eq. (7) to the value of B.

3. - Let us discuss another calculation of B [10] employing the finite energy sum rules for the two-point correlator

$$P(Q^2) = i \int \mathrm{dx}\ \exp[iqx] \langle 0|T\hat{O}(x)\hat{O}(0)|0\rangle$$

together with the method of effective chiral Lagrangians, the latter being used to fix the functional dependence of the corresponding spectral density on energy. The obtained value of B differs noticeably from both result (9) and the prediction of the vacuum saturation hypothesis. It should be noted that this calculation involved a large-size correction due to the operators m^2, m^4, $m\bar{q}q$. This fact forced the authors to use a rather large "duality interval" $S_0 = 8\,\mathrm{GeV}^2$ in handling their sum rules. On the other hand, within the approach of paper [12] all these contributions are exactly summed up in the factorized term. Note also that the use of the chiral perturbation theory assumes the independence of the form factor from the energy of the K^0 —$\bar{K}^0$ pair, which can hardly be a reasonable approximation at energies as large as $\sqrt{S_0} = (2.5 \div 3)\,\mathrm{GeV}$. The technicalities aside, we feel that the result of Pich and Rafael may well be somewhat underestimated.

4. - To conclude, we have used the x-space sum rules to get a direct estimation of the accuracy of the vacuum saturation technique for operators $\bar{u}\gamma^\mu d\bar{d}\gamma^\mu u$ and $\bar{u}\gamma^\mu\gamma_5 d\bar{d}\gamma^\mu\gamma_5 u$. Our result is that the v.e.v.'s of these operators are equal to each other with an accuracy of about 10%. This observation gives an additional support of the result of the calculation of the parameter B which was discussed in sect. 2.

References

[1] M.K. Gaillard and B.W. Lee, *Phys. Rev.* **D 10** (1974) 847.

[2] F.J. Gilman and M.A. Wise, *Phys. Lett.* **B 93** (1980) 129; *Phys. Rev.* **D 27** (1982) 1128.

[3] L. Wolfenstein, *Nucl. Phys.* **B 160** (1979) 501.

[4] R.E. Shrock and S.B. Treiman, *Phys. Rev.* **D 19** (1979) 2148.

[5] P. Golic, B.Guberina, J.Trampetic and D.Tadic, *Nucl. Phys.* **B 221** (1983) 141.

[6] J.F. Donoghue, E. Golowich and B.R. Holstein, *Phys. Lett.* **B 119** (1982) 412.

[7] B.Guberina, B. Machet and E. de Rafael, *Phys. Lett.* **B 128** (1983) 269.

[8] N. Cabibbo, G. Martinelli and R. Petronsio, *Nucl. Phys.* **B 244** (1984) 381.

[9] P. Cea, G. Nardulli and G. Preparata, *Phys. Lett.* **B 148** (1984) 477.

[10] A. Pich and E. de Rafael, *Phys. Lett.* **B 158** (1985) 477.

[11] P. Cea and G. Nardulli, *Phys. Lett.* **B 152** (1985) 251.

[12] K.G. Chetyrkin, A.L. Kataev, A.B. Krasulin and A.A. Pivovarov, *Phys. Lett.* **B 174** (1986) 104; hep-ph/0103230.

[13] K.G. Chetyrkin, S.G. Gorishny, A.B. Krasulin, S.A. Larin, and V.A. Matveev, preprint INRP-0337 (1984);
I.I. Balitsky and A.V. Yung, *Phys. Lett.* **B 114** (1982) 53;
B.L. Ioffe and A.V. Smilga, *Pis'ma Zh. Exp. Teor. Fiz.* **37** (1983) 250.

[14] K.G. Chetyrkin, N.V. Krasnikov and A.N. Tavkhelidze, *Phys. Lett.* **B 76** (1978) 83; N.V. Krasnikov and A.A. Pivovarov, *Phys. Lett.* **B 112** (1982) 397.

[15] V.M. Belyaev and B.L. Ioffe, *Zh. Exp. Teor. Fiz.* **83** (1982) 876.

[16] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B 147** (1979) 385.